

AN UPDATED THEORETICAL SCENARIO FOR GLOBULAR CLUSTER STARS

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ABSTRACT

In the first part of this paper we revisit the history of theoretical predictions for HB luminosities in old Population II stellar clusters, starting from the results of ‘old’ evolutionary computations to introduce in various steps all the available ‘new’ physics. We discuss the influence of physical ingredients on selected evolutionary parameters, finally presenting models which incorporate all the most recent updating of the relevant physics. The evolutionary behavior of such models is extensively investigated for selected choices about the cluster metallicity, discussing theoretical predictions concerning both cluster isochrones and the calibration of the parameter R in terms of the original amount of He in stellar matter. One finds that the ‘new’ physics has a relevant influence on both these parameters, moving in particular cluster ages to a much better agreement with current cosmological evaluations. This scenario is implemented by a further set of stellar models where element diffusion is taken into account. The comparison between theoretical scenarios with or without diffusion is presented and discussed. A discussion of current observational constraints to the light of the updated theory closes the paper.

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1. Introduction

Since galaxies were born in an already expanding Universe, the age of the Universe appears as a safe upper limit for the age of any star and any stellar cluster. The fact that several determinations of globular cluster ages yielded values larger than the age of the Universe as based on current evaluations of the Hubble constant (see, e.g., Van den Bergh 1994, Tanvir et al. 1995) has stimulated a renewed interest in the theory of globular cluster Pop. II stars. At the same time, significant improvements in the input physics needed for stellar evolution have been made, such that noticeable changes of the theoretical results could be expected. These improvements initially were motivated by the results of helioseismology, which opened a new window into the interior of the Sun, allowing for an extremely accurate determination of the solar structure. This provided a severe challenge for stellar structure theory. The efforts undertaken resulted in a new generation of opacity data (Rogers & Iglesias 1992; Seaton et al. 1994, Iglesias & Rogers 1996) and equations of state (Mihalas et al. 1990; Rogers et al. 1996), which led to a much better prediction of

solar oscillations and also resolved many long-standing problems in our understanding of pulsating stars. In addition, helioseismology required particle diffusion to be taken into account in solar models (see Bahcall et al. 1995 and references therein).

Stimulated by the success with regard to the Sun, the new opacities and equation of state, along with improvements in low-temperature opacities (e.g. Alexander & Fergusson 1994), nuclear cross-sections and neutrino emission rates, have been applied now to low-mass metal-poor stars in order to investigate the above-mentioned age problem. Several investigations (Chaboyer & Kim 1995; Mazzitelli et al. 1995: MDC; VandenBerg et al. 1996; D'Antona et al. 1997; Salaris et al. 1997: Paper I) have already shown that models with some of the updated physics inputs tend to predict lower cluster ages, thus at least decreasing the size of the discrepancy, if not resolving it. The new physics still needs to be applied to more massive and more metal-rich stars, although some of it, e.g. opacities, already are in use (Bono et al. 1997 a, 1997b, 1997c) However, the full consequences of all improvements have not yet been evaluated. As an example we mention the evolution and pulsations of Cepheid stars.

In the present paper we are concerned with Pop. II stars only. We intend to follow a twofold purpose. Firstly, we will present stellar models appropriate for globular cluster studies that include *all* of the improvements listed above. These models will cover the complete relevant mass and metallicity range, and include all evolutionary stages from the zero-age main sequence until the end of the helium-burning phase on the horizontal branch. Our calculations therefore provide the most up-to-date set of stellar models applicable to all problems of globular cluster dating. In particular, we show for the first time how particle diffusion influences the evolution of low-mass stars until the end of the horizontal-branch phase. In our feeling, in the future diffusion will become an integral part of modern stellar evolution, such as it already has been for several years in the case of solar models.

Secondly, we will demonstrate how each of the various steps in improving the input physics is influencing the models. This is important because of the variety of calculations available in the literature that include *some* but not all of the new physics. In order to compare these results, it is necessary to be able to translate the differences in physical assumptions into differences in stellar properties. In the first part of this paper we will approach this problem, starting from a suitable set of "old" evolutionary computations and introducing, step by step, the available "new" physics in order to make clear the influence of the new assumptions on selected evolutionary parameters. At the end of Sect. 2, we will finally present our best models which will incorporate all the most recent improvements in the relevant physics. However, these models will still be calculated ignoring element diffusion.

In Sect. 3 evolutionary predictions for these best models will then be extensively investigated for selected choices about the cluster metallicity, presenting theoretical predictions concerning cluster isochrones. This will be repeated in Sect. 4 for a set of stellar models where element diffusion is properly taken into account. The comparison between theoretical scenarios with and without diffusion will be presented and discussed. Section 5 will finally deal with a discussion of the influence on the R-parameter and the consequences for the inferred original amount of helium in stellar matter. We will also critically discuss the theoretical uncertainties on R. Finally, the conclusions follow.

2. Input physics and Pop.II models

As a starting point we will assume as a *reference* frame (step 1) the evolutionary scenario presented by Straniero & Chieffi (1991) and by Castellani et al. (1991: hereinafter CCP), which covers with a homogeneous set of computations the major evolutionary phases experienced by galactic globular cluster stars. As a relevant point, let us here recall that the above evolutionary scenario appears in excellent agreement with computations based on similar physics given by Sweigart (1987); in particular theoretical predictions concerning the mass of the He core at the He ignition agree within few thousandths of solar mass.

The ‘step 1’ column in Table 1 gives details on the relevant physics adopted in those models and which has been submitted to successive modifications. Top to bottom one finds:

- Equation of State (EOS) Str88: Straniero (1988) implemented at the lower temperature with Saha equation.
- Radiative Opacity for H, He mixtures (OPAC). LAOL: Los Alamos Opacity Library (Huebner et al. 1977) implemented at the lower temperature with Cox & Tabor (1976) opacity tables.
- Radiative Opacity for C, O mixtures (OPAC-CO). LAOL: Los Alamos Opacity Library (Huebner et al. 1977).
- He burning rates (α -rates). Fow75: Fowler et al. (1975), Harris et al. (1983), Caughlan et al. (1985).
- Neutrino energy losses (NEU). Mun85: Beaudet et al. (1967), Munakata et al. (1985), Richardson et al. (1982).

Electron screening (Graboske et al. 1973, DeWitt et al. (1973) and electron conductivity (Itoh et al. 1983) have not been submitted to relevant improvements since that time. As a matter of fact, numerical experiments performed with our code show that neither improvements in strong electron screening, as given by Itoh et al. (1977) and Itoh et al. (1979), nor the alternative approach to weak and intermediate screening (Mitler 1977)

do affect the evolutionary phases we are dealing with.

Table 1 gives a list of the various steps performed in the input physics together with the corresponding values for selected evolutionary quantities. The upper portion of the table gives the steps in updating the physics inputs, whereas in the lower portion of Table 1 one finds selected results concerning the H burning phase of a $0.8M_{\odot}$ model ($Y=0.23$, $Z=0.0001$) and the He burning phase of the same model but assuming the original mass reduced to $0.7M_{\odot}$ by mass loss. Top to bottom one finds: the luminosity (LogL^{TO}) and the age (t^{TO}) of the $0.8M_{\odot}$ H burning model at the track Turn Off (TO), the luminosity (LogL^{flash}), the age (t^{flash}) and the mass (M_c) of the He core at the He flash and the surface helium abundance (Y_{HB}) after the first dredge-up. For the He burning $0.7M_{\odot}$ one finally finds the Zero Age Horizontal Branch luminosity (LogL_{ZAHB}), and effective temperature (LogTe_{ZAHB}) together with the time t_{HB} spent in the central He burning phase as a Horizontal Branch (HB) star. Here, as well as through all this paper, luminosities and masses are in solar units.

As shown in Table 1, the updating of the input physics runs as follows:

- i) EOS from Str88 to OPAL (Rogers 1994, Rogers et al. 1996), implemented in the temperature-density region not covered by OPAL with Str88, plus Saha EOS in the outer stellar layers. The transition from OPAL to other EOS appears smooth and without discontinuities.
- ii) OPAC and OPAC-CO from LAOL to OPAL (Rogers & Iglesias 1992, Iglesias & Rogers 1996),
- iii) α -rates from Fow75 to Cau88 (Caughlan & Fowler 1988) and, finally,
- iv) NEU from Mun85 to Haft94 (Haft et al. 1994) for plasma neutrino production and Itoh et al. (1996) for the other kinds of neutrino energy losses.

Even a quick inspection of results in Table 1 shows the relevant effects produced by the OPAL-EOS on the MS lifetimes and TO-luminosities, an occurrence already well discussed in the literature (see, e.g., Chaboyer & Kim 1995). For HB models, one finds that improvements in the opacity of H-rich mixtures have the major effect of moderately increasing the HB luminosity ($\Delta\text{LogL} \sim 0.02$) and decreasing the HB lifetime by 3.4 %. As expected, CO opacity affects only the advanced phases of central He burning, decreasing the HB lifetimes by a further 7 %. As a whole, one finds that the major effect of the new opacities is the decrease of HB lifetimes by the not negligible amount of about 10 %. Step 4 in Table 1 shows that the passing from the previous EOS to the more recent OPAL EOS does not affect HB lifetimes; however one finds that the HB luminosity increases by a

further $\Delta \text{LogL} \sim 0.02$, in spite of the small decreases in M_c , whereas the age of the flashing RG decreases by about 2 Gyr.

Steps 5, 6, 7 and 8 finally report the effect of improved evaluations of the triple α nuclear reactions and of the plasma neutrino energy loss rates. On very general ground, one expects that both these mechanisms affects the He ignition at the flash, affecting in turn the structure of the initial ZAHB models. To disentangle this effect from the effect on the physics of HB models, step 5 and 6 concern only ZAHB models, introducing the new rates for 3α reactions (Caughlan & Fowler 1988) and for plasma neutrino production (Haft et al. 1994) in two subsequent steps *for the fixed value of the ZAHB Helium core mass* given by the result of step 4. One finds that the new 3α rates further increase, though slightly, the HB luminosity, whereas HB lifetimes are again substantially decreased by a further 8%. On the contrary, one finds that HB structures are only marginally affected by the NEU treatment, as early predicted (Gross 1973).

Step 7 shows the effect of new 3α rates on H burning models as HB progenitors. Step 8 gives finally the results for our ‘best’ models where all the available updating of the physics have been taken into account. Due to the effect of both 3α rates and NEU, the He flash is delayed and the top luminosity of the RG structures is increased, becoming about 0.2 mag brighter than in Straniero & Chieffi (1991; step 1 in Table 1). Correspondingly the value of M_c ‘jumps’ from $0.5054 M_\odot$ to $0.5152 M_\odot$, contributing to a further increase of the HB luminosity. From data in Table 1, one recognizes that 3α rates and NEU give a similar contribution to the quoted increase of M_c . As a whole, one finds that passing from CCP to present best models the major modifications about HB evolution appear an increase of the ZAHB luminosity by about $\Delta \text{LogL} \sim 0.06$ (~ 0.15 mag.) whereas HB lifetimes are decreased by the huge amount of, about, 23 %. As one can easily understand, and as we will discuss later on, this will have rather dramatic effects on current calibration of the R parameter.

To orientate the reader in the current literature, let us review available theoretical estimates in terms of the quoted physical scenarios. As a starting point, let us notice that CCP computations adopt more or less improved input physics than adopted in previous computations (as, e.g., Sweigart 1987, Dorman & VandenBerg 1989, Lee & Demarque 1990). Dorman (1992) adopts neutrino energy losses and opacities as in CCP, improving nuclear reactions rates as in Caughlan & Fowler (1988) but taking the EOS from Eggleton et al. (1973). Dorman et al. (1993) adopt the same inputs as Dorman (1992), but low-temperature opacities from Alexander (1975, 1981). Mazzitelli et al. (1995) have OPAL EOS and opacity, but using Dappen et al. (1988) EOS in H burning models (as stated in D’Antona et al. 1997 who updated the turn off models with OPAL EOS); nuclear reactions rates are from Harris et al. (1993) and neutrinos from Itoh et al. (1989). Salaris et al.

(1997) models overlap present step 4 assumptions. As a result, one finds that our step 8 is till present the only one including all available updating of the input physics. According to such an evidence, in the following section we will investigate the evolutionary behavior of similar models, discussing the calibration of the most relevant evolutionary parameters.

3. ‘Best’ canonical models

The evolutionary behavior of our ‘best’ models, as defined by step 8 in the previous section, has been investigated for selected choices on the assumed star metallicity and adopting everywhere an original amount of He given by $Y=0.23$ as a suitable value for population II stars. In all cases we assumed solar scaled composition as given by Grevesse & Noels (1993). However, alpha-enhanced distributions can be taken into account bearing in mind the scaling law discussed by Salaris et al. (1993). It is worth noting that the validity of such relation has been recently questioned by VandenBerg & Irwin (1997), but for a metal-rich regime ($[Fe/H] > -0.8$) and for large α -enhancement factors ($[\alpha/Fe] > 0.3$; see also Weiss, Peletier & Matteucci, 1993, for the same topic), i.e., for values beyond the range suitable for globular cluster stars.

Table 2 gives selected data of the models at the track turn-off (TO) for the various choices on the stellar mass and for the metallicity $Z=0.0001, 0.0002, 0.001$ and 0.006 . Left to right one finds: the metallicity (Z), the mass of the model (M), the age (t^{TO}), the luminosity ($\log L^{TO}$) and the effective temperature ($\log T_e^{TO}$) at the track Turn Off (TO).

On the basis of these evolutionary tracks H burning isochrones have been computed for the quoted assumed metallicities and covering the range of ages suitable for galactic globular cluster stars. Table 3 gives detailed informations on the isochrone TO luminosity and effective temperature. Left to right one finds: the metallicity (Z), the age (t^{TO}), the luminosity ($\log L^{TO}$), the effective temperature ($\log T_e^{TO}$) and the mass of the model (M^{TO}) at the isochrone Turn Off (TO). As expected, data for the case $Z=0.0002$ overlap similar computations presented in Paper I, since passing from step 4 to step 8 affects only the advanced evolution of RG and HB structures. Thus present computations may be regarded as an extension to larger metallicities of the quoted computations.

We agree with the comment of our unknown referee about the risk of using TO luminosity as a parameter to derive cluster ages. From an observational point of view it appears quite difficult to define this parameter with high accuracy (Richer, Fahlman & VandenBerg 1988); the average uncertainty on the TO magnitude can be estimated of the order of $\simeq \pm 0.10$ mag, which leads to an indetermination on the derived age of the order

of ± 1.5 Gyr (see Chaboyer, Demarque & Sarajedini 1996 for a discussion on this point). Moreover, from a theoretical point of view, being the TO defined as the bluest point on the isochrone, the determination of the TO magnitude $M_V(TO)$ is related to the isochrone color (Chaboyer 1995, Chaboyer et al. 1996), thus depending on the stellar effective temperature, i.e., on the stellar radius, which can be affected by significant uncertainties, depending on the theoretical treatment of convection in superadiabatic layers (MDC). For such reasons, the use of other age indicators (see, e.g., Chaboyer et al. 1996) has been suggested in several investigations. However, since such a parameter is still widely in use, let us discuss with some details our results on the matter, at least to allow a comparison with previous results appeared in the literature.

The best fit of the data for the dependence of the TO luminosity on the cluster ages gives the analytical relations:

$$\text{Log}t_9 = -0.946 \text{ Log}L^{TO} + 1.465 \text{ (Z=0.0002).}$$

$$\text{Log}t_9 = -1.117 \text{ Log}L^{TO} + 1.414 \text{ (Z=0.001).}$$

$$\text{Log}t_9 = -1.239 \text{ Log}L^{TO} + 1.325 \text{ (Z=0.006).}$$

where t_9 is the cluster age in Gyr. These relations reproduce the computational results in the range 8 to 18 Gyr with a maximum error of few 10^8 years. As a result, one finds that, for each given age, the TO luminosities are predicted to decrease with metallicity with a slope in fair agreement with a large body of previous predictions but with lower predicted luminosities for each given age. This is shown in Fig. 1, which compares the dependence of present TO luminosities on the assumed metallicity for a given age ($t=12$ Gyr) with similar results already appeared in the literature. In the figure (as well as in some other following figures) theoretical expectations for O-enhanced mixtures (Bergbusch & VandenBerg 1992, VandenBerg 1992, Dorman et al. 1993) are reported versus the total fraction of heavy elements, as obtained by the same quoted papers.

One finds that present results represent in all cases a lower boundary for current evaluations of TO luminosities, thus decreasing current age estimates for a given TO luminosity. More in details, present results predict TO luminosities systematically lower by about $\Delta \text{Log}L \simeq 0.05$ with respect to previous computations, but Mazzitelli, D’Antona & Caloi (1995: MDC) who, independently of the adopted treatment of superadiabatic convections, predict again larger luminosities but with a difference from present results

which decreases as the metallicity increases. According to the previous analytical relations, one easily finds that the quoted decrease $\Delta \text{LogL} \simeq 0.05$ implies a corresponding decrease by about 10% in previous estimates of globular cluster ages. We will further discuss this point in the final section.

Beyond the problem of TO luminosities, H burning models deserve further attention as progenitors of He burning models, determining the structural parameters which will constrain the evolutionary behavior and, in particular, the luminosity of HB stars. Computational results concerning those parameters are reported in Table 4 for the labeled assumptions about the cluster metallicity. Left to right one finds: the metallicity (Z), the mass (M_c) of the He core at the He flash, the surface helium abundance (Y_{HB}) after the first dredge-up, the age (t^{flash}) and the luminosity (LogL^{flash}) at the He flash and the mean value between the minimum and the maximum in luminosity (LogL^{bump}) during the RGB ‘bump’. Data in Table 4 will allow the approach of He burning phases adopting self-consistent evolutionary values for the two parameters characterizing a ZAHB structure, namely the mass of the He core (M_c) and the He abundance in the stellar envelope. Since both values depend only marginally on the assumptions made about the cluster ages, Table 4 reports the values corresponding to a $0.8 M_\odot$ evolving Red Giant which can be safely assumed as representative of theoretical expectations in a sufficiently large range of ages. Here let us notice that the discussed increase (see Table 1) of the predicted luminosity of the RG tip would affect current estimate of the Hubble constant H_0 when using such a feature as a distance indicator. As a matter of the fact, one easily finds that the quoted increase by 0.2 mag in the top RG luminosity implies an increase by about 10% in the distance and, in turn, a decrease by the same amount of the H_0 estimate.

The amount of extra He (ΔY) brought to the stellar surface by the first dredge up appears in good agreement with similar evaluations already given in the literature (see, e.g., Castellani & Degl’Innocenti 1995 and references therein). Figure 2 compares present masses of the He cores in the flashing Red Giants with previous results on that matter. Again one finds that all current evaluations, but MDC, have a rather similar dependence on the assumed metallicity. However, one finds that our ‘best’ models predict M_c values in all cases larger than previous predictions, an occurrence which acts in the sense of increasing the expected luminosity of ZAHB structures.

By adopting M_c and ΔY values from H burning models we are now in the position of predicting the evolutionary behavior of He burning Horizontal Branch (HB) structures. Table 5 gives detailed informations on the HR diagram location of Zero Age Horizontal Branch (ZAHB) together with a comparison between present and CCP He burning lifetimes. Left to right one finds: the mass (M), the luminosity (LogL) and the effective temperature

($\log T_e$) of the zero-age horizontal-branch, ZAHB, model (following CCP we assume as ZAHB structures the models already evolved by 1 Myr), the time (t_{He}) spent during the central He burning (until the disappearance of the convective core) and the same quantity (t_{He}^{CCP}) but for the CCP models.

As expected on the basis of the exploratory computations given in the first part of this paper, one finds that the expected luminosity of ZAHB models is substantially increased whereas He burning lifetimes in all cases fall down by more than 20%. Figure 3 presents predictions about the luminosity of the ZAHB model in the RR Lyrae instability strip ($\log T_e=3.85$) with previous results on that matter. One finds that ‘old’ computations, as given by CCP or Lee & Demarque (1990) agree in predicting lower luminosities, by about $\Delta \log L \approx 0.05$. This occurrence implies that, when using ZAHB models as ‘standard candles’ to constrain the cluster distance modulus (DM), ‘old’ computations would produce smaller DM, thus lower luminosities of the observed TO and, finally, larger ages. The same figure shows that all the more recent computations agree in predicting more luminous ZAHBs. In particular one finds that at the lowest metallicity, we predict luminosities in close agreement with MDC, notwithstanding the (small) difference in the He core masses. Note that the difference at the larger metallicities can be understood in terms of the different slope of the M_c -metallicity relation already disclosed in Fig. 2.

Figure 4 finally compares present He-burning lifetimes with the ones given in CCP and with the value originally predicted by Buzzoni et al. (1983) for the two assumed metallicities. The emerging scenario concerning current evaluation of the amount of original He in globular cluster stars will be discussed in Sect. 5.

4. Element diffusion

A general discussion on the relevance of element diffusion in the evolution of Pop.II stars has been already given in a previous paper (Castellani et al. 1997: Paper II) and it will be not repeated here. Here we only recall that the diffusion of both He and heavy elements is taken into account, according to the algorithm adopted in Thoul et al. (1994). Table 6 gives selected evolutionary properties of models for the various choices on the stellar mass and for the investigated metallicities. As a whole, our results closely follows the general trends discussed by Proffitt & Vandenberg (1991) in their pioneering paper to which we address the reader for a general discussion. Figure 5 compares the run in the HR diagram of isochrones with and without element diffusion for a selected metallicity and for the labeled choices on the cluster age whereas Table 7 gives detailed informations on the isochrone TO luminosity and effective temperature. Tables 6 and 7 are the homologous of

Tables 2 and 3 given in the previous section for computations neglecting element diffusion. Figure 6 shows the dependence of isochrones TO luminosity on cluster age, as compared with similar results but without allowing for element sedimentation.

In the case of sedimentation the best fitting of the data connecting ages to TO luminosities gives for ages between 8 and 18 Gyr the relations:

$$\text{Logt}_9 = -0.924 \text{ LogL}^{TO} + 1.414 \text{ (Z=0.0002)}$$

$$\text{Logt}_9 = -1.130 \text{ LogL}^{TO} + 1.384 \text{ (Z=0.001)}$$

$$\text{Logt}_9 = -1.170 \text{ LogL}^{TO} + 1.288 \text{ (Z=0.006)}$$

which again reproduce the results within a few 10^8 years.

According to the discussion given in Paper II for the case Z=0.0004, one finds that element diffusion moderately decreases theoretical expectations for TO luminosities for each given cluster age. Figure 6 now shows that such an effect depends on the assumed metallicity, increasing when the metallicity is decreased. As a matter of the fact, the effect of diffusion on the TO luminosity is larger in the most metal poor isochrones since the diffusion in the stellar envelopes is larger due to the thinner convective envelopes on MS stars. At the lowest metallicity (Z=0.0001) $\Delta \text{LogL} \approx 0.04$: thus, for a given observed TO luminosity, allowing for sedimentation would decrease age estimates by about 10%. On the contrary, when Z=0.006 one expects rather negligible variations.

Table 8 gives selected structural parameters of models at the He flash, to be used as inputs to the He-burning models (for a discussion of the effects of diffusion on the luminosity of the RGB ‘bump’, see also Cassisi et al. (1997). On this basis we present in Fig. 7 a comparison between HB evolution with or without diffusion; in the same figure the path in the HR diagram of these models is also compared with similar results from CCP. Table 9 gives details on the ZAHB structures and on the corresponding helium burning evolutionary times. One has to advice that HB structures presented in both Fig. 7 and Table 9 assume a $0.8 M_{\odot}$ model as H-burning progenitor. However, Table 8 shows that increasing, e.g., the RG masses (thus decreasing the cluster age) M_c decreases but Y_{HB} increases, with balancing effects on the predicted HB luminosities. As a result, numerical experiments disclosed that HB data based on a $0.8 M_{\odot}$ progenitor can be safely taken as representative of HB models in the range $0.7 < M/M_{\odot} < 0.9$, at least, thus covering quite

a large range of cluster ages.

Figure 8 summarizes the results of this paper relevant for the problem of cluster ages, disclosing the run with metallicities of ZAHB and TO luminosities, with or without diffusion, and for selected choices about the assumed ages. In this figure, data concerning the TO luminosity have been implemented with similar data but for $Z=0.0004$, as given in Paper II with a ‘step 4’ physics which is fully compatible with present computation as far central H burning models are concerned. As already discussed, present HB are brighter than estimated in Paper II (see Table 1). However, the same Fig. 8 shows that present computations keep predicting that diffusion decreases the HB luminosity by about $\Delta \text{LogL} \sim 0.02$, in agreement with the results of Paper II.

Figure 9 shows the calibration of age in terms of the difference in luminosities between ZAHB (taken at $\text{LogTe}=3.85$) and TO, as predicted with or without diffusion, and as compared with original predictions in CCP. As discussed in Paper II one finds that diffusion plays a minor role in that calibration. However, the same Fig. 9 shows that the new physics, as a whole, reduces by about 4 Gyr theoretical calibrations based on the old physics.

The detailed comparison between theory and observation is a delicate question, beyond the purpose of this theoretical paper. However, one may test present predictions vis-a-vis recent estimates of HB luminosities derived by recent Hipparcos parallaxes measurements. This is shown in Fig. 10, which compares the data presented by De Boer et al. (1997) with our theoretical predictions converted in M_V , $B-V$ magnitudes according to model atmospheres by Kurucz (1992). One finds that the HB luminosity level appears in quite good agreement with the quoted observations. Here we only notice that the two stars which lie below the ZAHB around $B-V \simeq 0$ both have been corrected for a rather large reddening ($E(B-V)=0.10$), one lacking -to our knowledge- of recent metallicity estimates.

As for the outcome of the improved theoretical scenario, let us recall that in Paper I it has been already shown that in the step 4 scenario the color magnitude diagram of a typical metal poor galactic globular can be reproduced by a 12 Gyr (no diffusion) isochrone. Taking into account that, at $\text{LogTe}=3.85$, our best HB models without diffusion turn out to be more luminous by $\Delta \text{LogL}_{ZAHB} \approx 0.02$, one estimates that the ‘new’ age shifts toward 11 Gyr without sedimentation, and even below if sedimentation is taken into account.

Figure 11 discloses that present results foresee a rather low dependence of $\Delta \text{LogL}(\text{HB-TO})$ on the cluster metallicity. As a matter of the fact, assuming, e.g. an age of 12 Gyr one finds that passing from $Z=0.0002$ ($[\text{Fe}/\text{H}]=-1.97$) to $Z=0.001$ ($[\text{Fe}/\text{H}]=-1.27$) we predict an increase in $\Delta \text{LogL}(\text{HB-TO})$ corresponding to ≈ 0.08 mag, independently of any assumptions about the efficiency of sedimentation, this difference increasing when the cluster age is

decreased.

5. The parameter R

Since the pioneering paper by Iben (1968) one knows that evolutionary predictions on the evolution of Pop.II stars can be used to constrain the amount of original He in globular cluster stars. Calibrations of the R parameter, i.e., the number ratio between HB stars and RG more luminous than the *HB luminosity level* have been given by Buzzoni et al. (1983) and, more recently, by Caputo, Martinez Roger & Paez (1987) and by Bono et al. (1995). According to current estimates, observational values for R appear ranging around $R \sim 1.1$. In terms of the quoted calibrations this implies $Y \sim 0.23$, which is consequently the value currently adopted in discussing globular cluster stars.

However, the evolutionary results discussed in the previous sections deeply affects such a scenario. We already found that the updated physics moderately affected (increased) theoretical expectations about HB luminosities, largely decreasing HB lifetimes. According to such an evidence, one expects a decreasing value of R and -thus- a larger predicted value of Y for any given value of R . Owing to the relevance of the argument, let us derive a quantitative evaluation of R as given by updated predictions about evolutionary times both along the RG and through the HB evolutionary phases. It has been already found that evolutionary times along the upper portion of the RG branch show a rather negligible dependence on both the chemical composition (within Pop.II limits) and mass of the evolving stars (see e.g. Castellani & Castellani 1993, Bono et al. 1995, Salaris & Cassisi 1997). Now we find a small but not negligible dependence on the efficiency of sedimentation. By best fitting computational results we find in the interval $1.5 \leq \text{LogL} \leq 1.8$:

$$t^{flash-t} = 730.93 - 629.14 \text{ LogL} + 144.73 (\text{LogL})^2 \text{ (No Diffusion)}$$

$$t^{flash-t} = 732.93 - 625.73 \text{ LogL} + 143.32 (\text{LogL})^2 \text{ (Diffusion)}$$

where $t^{flash-t}$ represents the time (in 10^6 yr) spent by a RG above the luminosity L . However, when $Z=0.006$ these relations can be safely used only in clusters with age lower than, about, 13 Gyr. At larger ages, the clump of stars along the RG branch becomes fainter than the HB luminosity level, as disclosed by data in the previous Table 8, and the relations would require a correction to properly account for such an occurrence (see Bono et al. 1995 for a discussion on that matter).

According to the procedure envisaged by Bono et al. (1995) we will take as reference the luminosity level of the ZAHB at $\text{LogTe}=3.83$, evaluating the time spent by RG stars above such a luminosity and taking HB evolutionary lifetimes from the models starting HB evolution at that effective temperature. Table 10 gives data about these two ingredients together with the corresponding estimates of R for the labeled choices on the metallicity, with or without allowing for the efficiency of sedimentation. Top to bottom one finds: the luminosity ($\text{LogL}_{\text{ZAHB}}$) of the ZAHB model at $\text{LogTe}=3.83$ the time (t_{He}) spent by the same model during the central He burning (until the disappearance of the convective core), the time (t_{RG}) spent by RGB stars above $\text{LogL}_{\text{ZAHB}}$, the value ($R(3.83)$) of the corresponding R parameter and the same values but when the ZAHB luminosity level is artificially increased by $\Delta\text{LogL}=0.05$ ($R(3.83)+0.05$) and 0.1 ($R(3.83)+0.1$). As already known, one recognizes that an increase of the metallicity tends to slightly increase the expectations on R for a given value of Y . Focusing, e.g., our attention on the case $Z=0.001$, one finds that when $Y=0.23$ the theoretical prediction given by Bono et al. (1995), $R=1.19$, should now be decreased to $R=1.05$ for model without sedimentation or to $R=0.95$ if sedimentation is taken into account. According to all available calibrations of R one finds $\delta Y \approx 0.4 \delta \text{LogR}$. As a consequence, present evolutionary scenario would predict that current estimate of original He should be increased by about $\Delta Y \sim 0.02$ if sedimentation is neglected, or by about $\Delta Y \sim 0.04$ with sedimentation at work. As a result, observational data already interpreted in the literature as an evidence for $Y=0.23$ should now lead to the rather unpalatable conclusion $Y \simeq 0.27$.

However, before entering in a discussion of the values in Table 10, one has to notice that the calibration of R depends on He-burning evolutionary times which, in turns, are mainly governed by the poorly determined cross section for $^{12}\text{C} + \alpha$ reaction (see also Dorman 1992). Along this paper we adopted for He burning reactions the rates given by Caughlan & Fowler (1988) which should improve previous evaluations given by the same authors in 1985. Comparison between these two rates shows a rather negligible differences in the triple alpha rates, but a large decrease in the $^{12}\text{C} + \alpha$ rates which, in turn, largely contributes to the decrease of HB lifetimes one finds in Table 1 between steps 4 and 7. As a matter of fact, about 60% of this decrease in HB lifetime (and of the corresponding decrease in the predicted value of R) can be attributed to these new rates. However, errors estimates on such a cross section are still as large as a factor of two, containing in this range also previous estimates given by Caughlan et al. (1985). Moreover, numerical experiments performed on HB models adopting recent reaction rates by Buchmann (1996), with errors estimates still of about 70%, tends to move the lifetimes toward the values estimated in old computations, based on Caughlan et al. (1985). One can only conclude that theoretical calibrations of R in terms of Y are still affected by too large errors to be used for accurate

calibrations of such a relevant parameter, and that the values of R given in Table 10 are still affected by theoretical errors corresponding to an error on Y of about $\Delta Y \sim 0.02-0.03$. If one adds the further errors related to the observational procedure, i.e., the errors on the HB luminosity level, on the bolometric correction for the corresponding RG stars and on the star counts (see, e.g., Brocato et al. 1995) one should conclude that R still appears as a too risky parameter to allow evaluations of Y with a reasonable accuracy.

Last two rows in Table 10 finally give theoretical evaluations for R when the adopted luminosity level is artificially increased above the ZAHB level by $\Delta \text{Log}L=0.05$ and 0.1, respectively. These values can be used to evaluate theoretical expectations on R when the mean luminosity of RR Lyrae is taken instead of the ZAHB luminosity as reference luminosity level. In the meantime these values give an estimate of the error on Y produced by observational errors in that level. One easily finds that an overestimate by $\Delta \text{Log}L=0.05$ (0.125 mag.) will produce an overestimate of He by about $\Delta Y \sim 0.015$. Note that previous evaluations of R appear only as a lower limit for theoretical expectations for cluster with blue HB. Less massive, hot HB structures have He burning evolutionary times increased by 20% or more (see Fig. 7 and Castellani et al. 1994), with a corresponding increase on theoretical expectations about the parameter R .

6. Conclusions.

In this paper we have followed the evolution of theoretical predictions concerning Pop.II stellar models vis-a-vis the recent progresses in the input physics. Stellar models including all the more recent evaluations of theoretical ingredients have been presented and discussed, with particular regard to the problem of globular cluster ages. We found that similar models tend to decrease previous estimates about the cluster age. The account for element sedimentation goes in the same direction. As a whole one finds that ‘canonical age estimates’, as given in CCP, have to be decreased by about 4 Gyr, promising a much better agreement with cosmological constraints. We finally drove the attention on the large indetermination of the theoretical procedure adopted to constrain the cluster original abundance of He, concluding that accurate results on that matter must wait for a better determination of the nuclear cross section $^{12}\text{C} + \alpha$.

Detailed tabulations on both evolutionary tracks and/or cluster isochrones are available upon request by E-mail.

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7. Figure caption

Fig. 1.— Behavior of the TO luminosity on the assumed metallicity for a given age ($t=12$ Gyr). Results for present ‘best’ canonical models are compared with similar results available in the literature. For the MDC 1995 models CM indicates the adoption by the authors of the Canuto & Mazzitelli (1991) treatment of overadiabatic convection while MLT indicates the adoption of the usual mixing lenght theory.

Fig. 2.— He core masses at the He flash as a function of metallicity for present models (canonical and with element diffusion) as compared with similar data already appeared in the literature.

Fig. 3.— The ZAHB luminosity at $\log T_e = 3.85$, as a function of metallicity for present models, compared with previous results, as labeled.

Fig. 4.— Central He-burning lifetimes as a function of the ZAHB effective temperature for present models (solid line) compared with similar data in CCP (dashed line) and with the predictions by Buzzoni et al. (1983) for HB models with $\log T_e = 3.83$ (stars). Metallicities as labeled.

Fig. 5.— H burning isochrones for $Y=0.23$, $Z=0.001$ and for the labeled ages for the present models without and with element diffusion (upper and lower panel, respectively). The time interval between consecutive isochrones is 1 Gyr. Note that the standard isochrones are calculated until a luminosity lower than that of the helium flash.

Fig. 6.— Dependence of the TO luminosity on the cluster ages for the three labeled metallicities. The results for canonical models (dashed line) are compared with similar results but for models with element sedimentations (solid line).

Fig. 7.— Comparison of the HB evolution for models with (solid line) and without (dashed line) diffusion for $Z=0.001$ and $Y=0.23$. Similar results from CCP (dot-dashed line) are also shown.

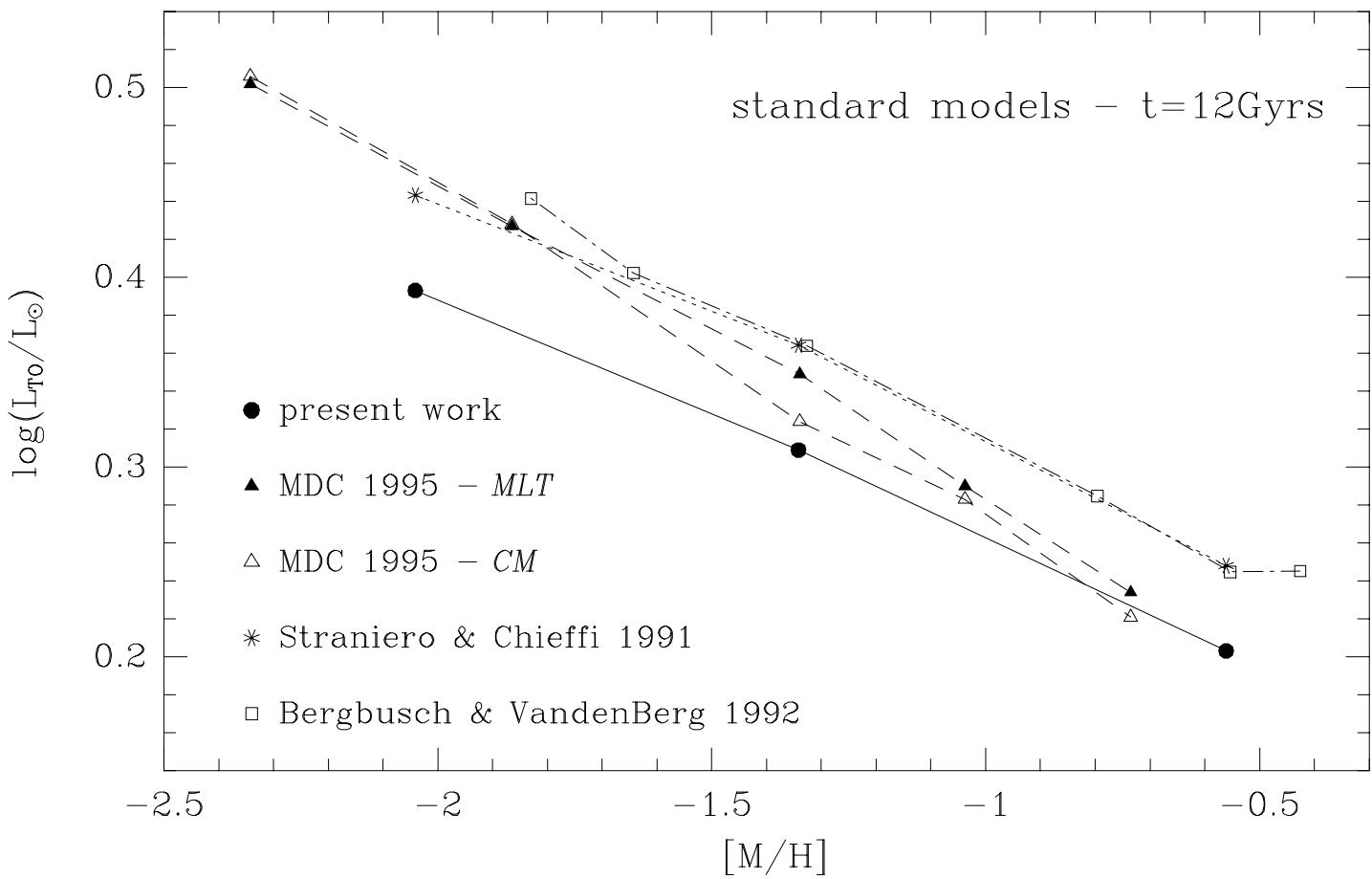
Fig. 8.— TO luminosities for selected labeled ages and ZAHB luminosities at $\log T_e = 3.85$ as a function of metallicity, for models with (solid line) and without (dashed line) element diffusion.

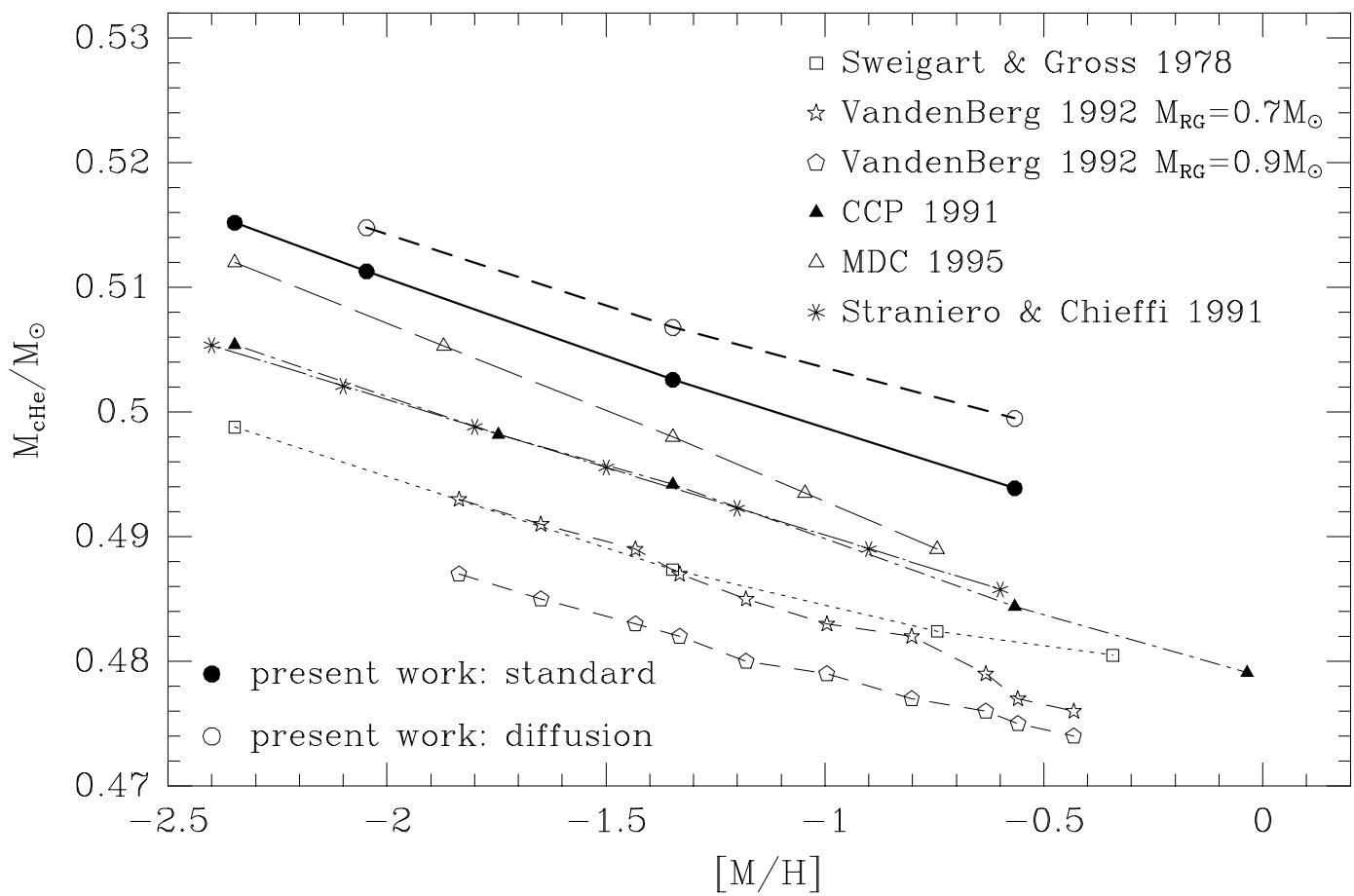
Fig. 9.— The calibration of age in terms of the difference in luminosities ($\Delta \log L(\text{HB} - \text{TO})$) between ZAHB (at $\log T_e = 3.85$) and TO, as predicted by present models with (solid line) and without diffusion (dashed line) and $Z=0.0002$. The results are compared with original

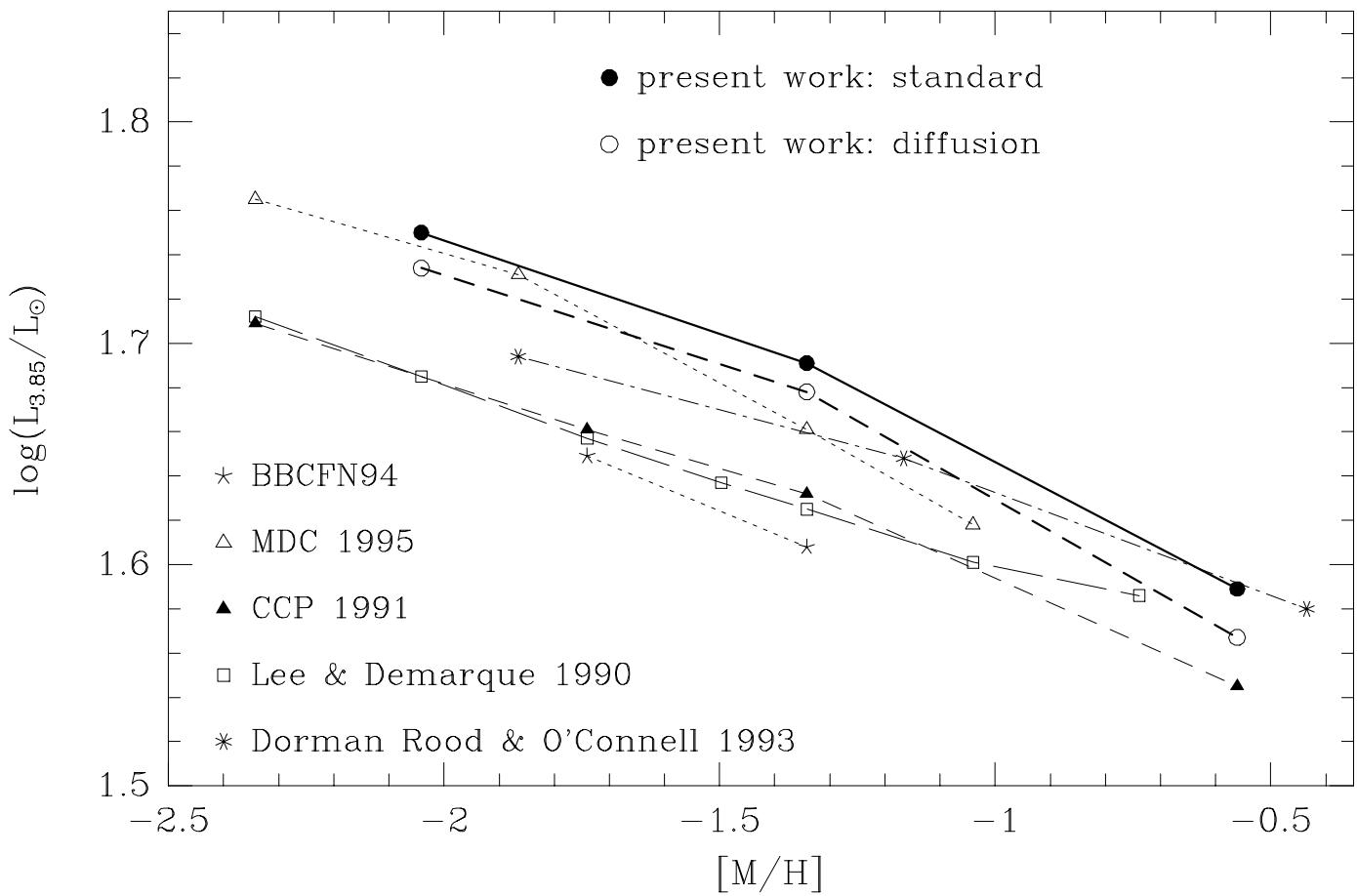
predictions by CCP (dotted line).

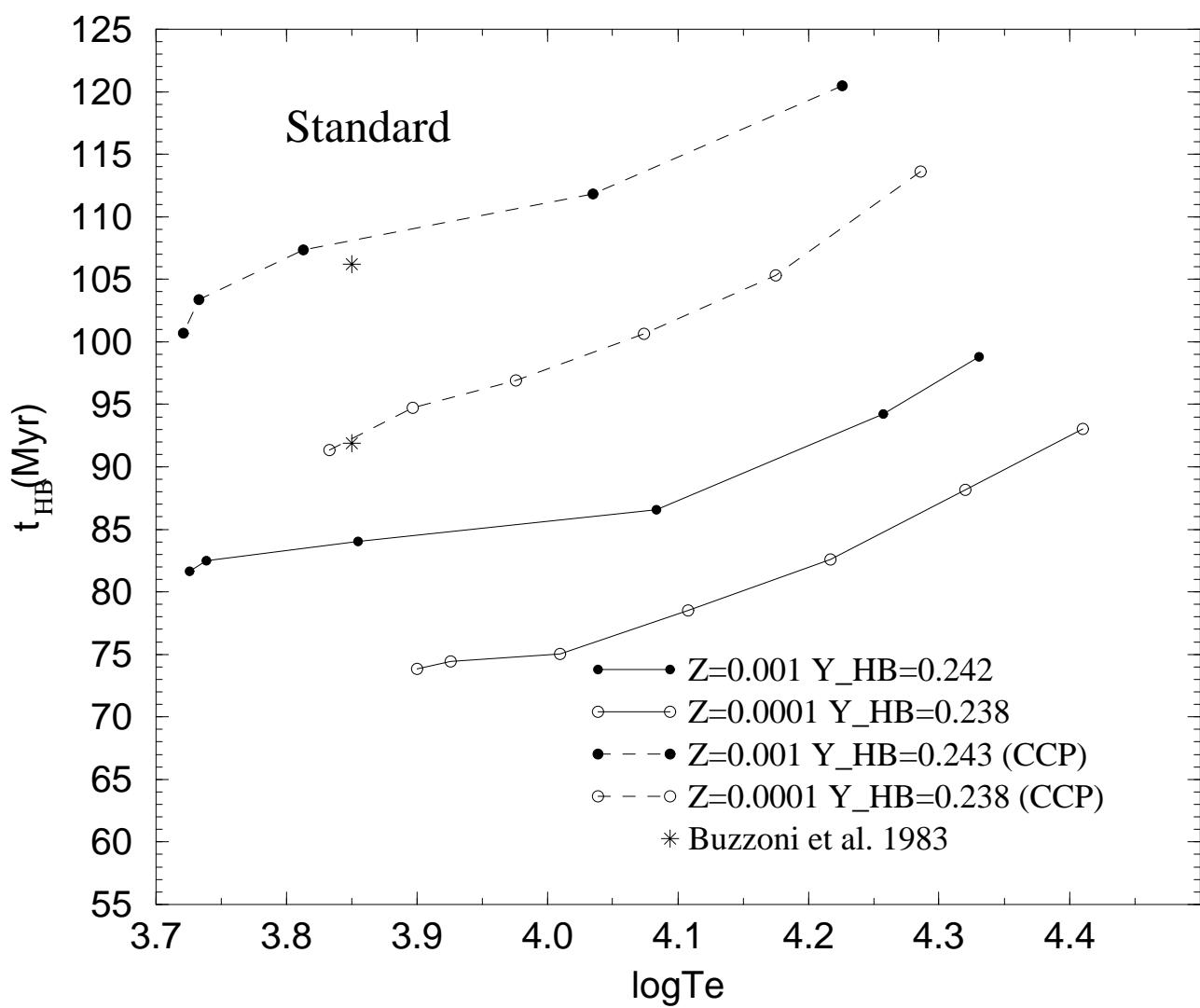
Fig. 10.— *a*) Theoretical ZAHB for standard models compared with Hipparcos estimates of HB magnitudes from De Boer et al. 1997 (see text). When available, labeled metallicities are from Gray et al. 1996. *b*) As *a*) but for models computed by accounting for element diffusion.

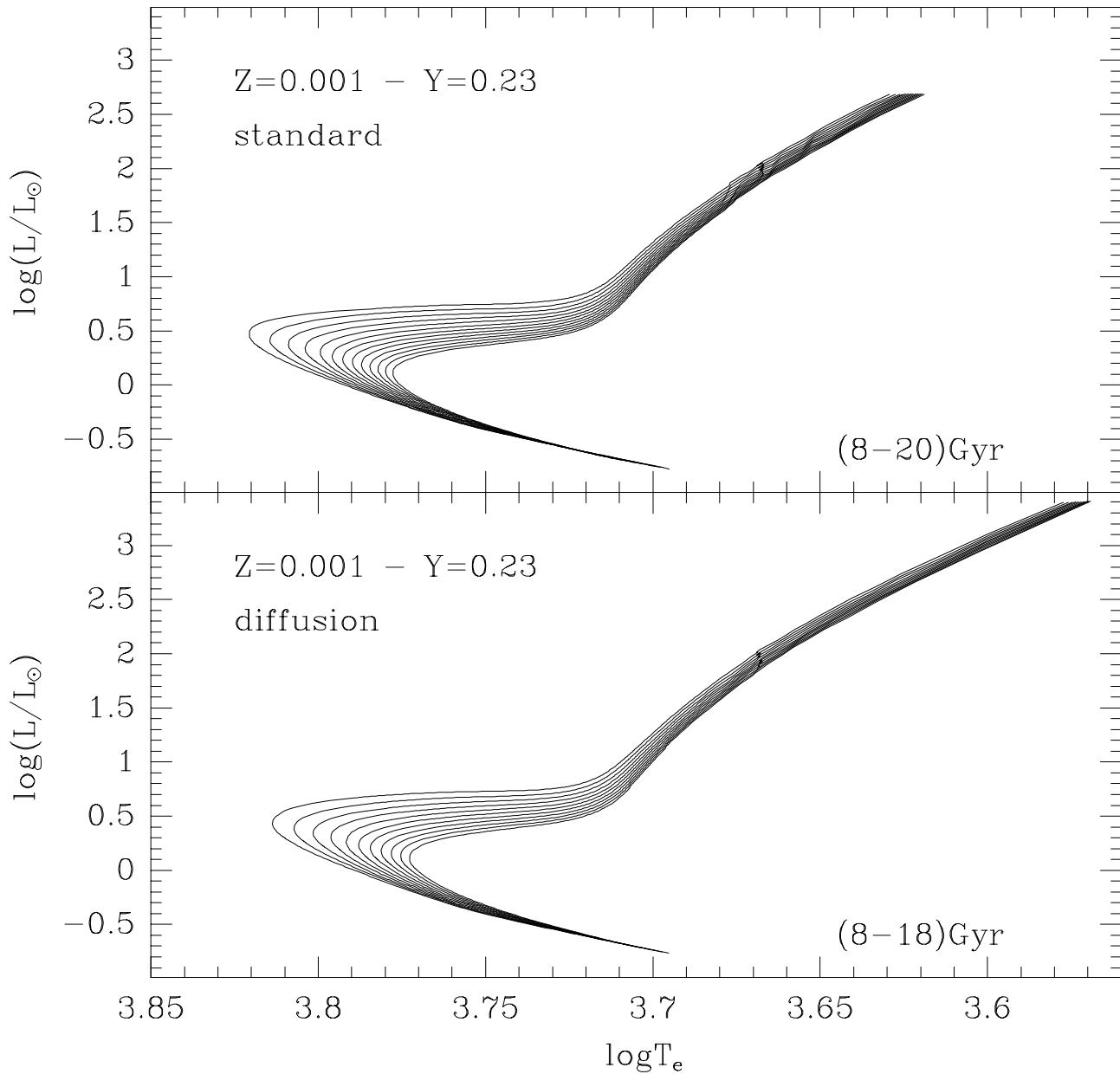
Fig. 11.— The dependence on metallicity of the difference in luminosities between ZAHB (at $\text{LogTe}=3.85$) and TO ($\Delta\text{LogL}(\text{HB-TO})$), as predicted by present models with (solid line) or without diffusion (dashed line) for selected labelled ages. Present results are compared with original predictions in CCP (dotted line).

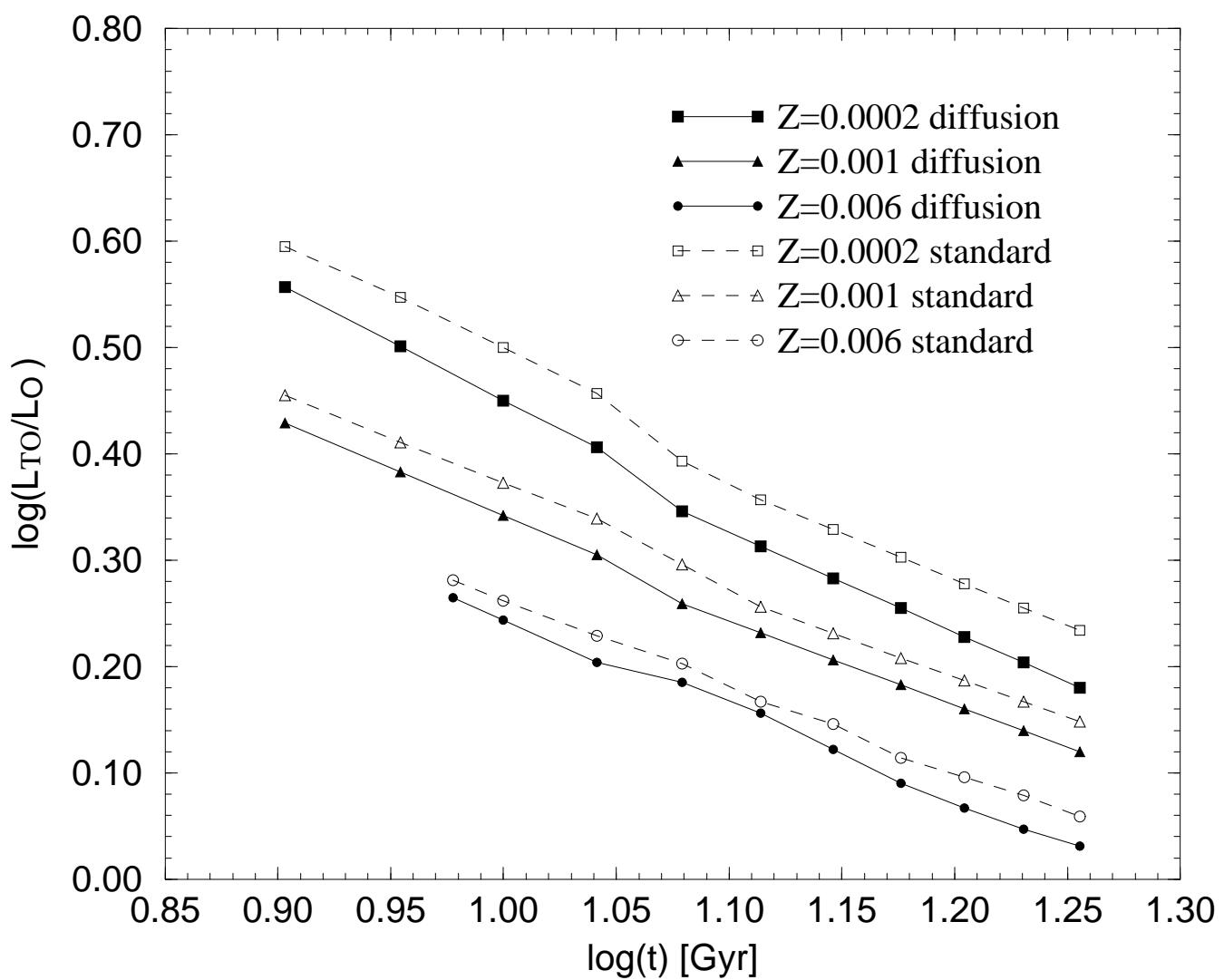


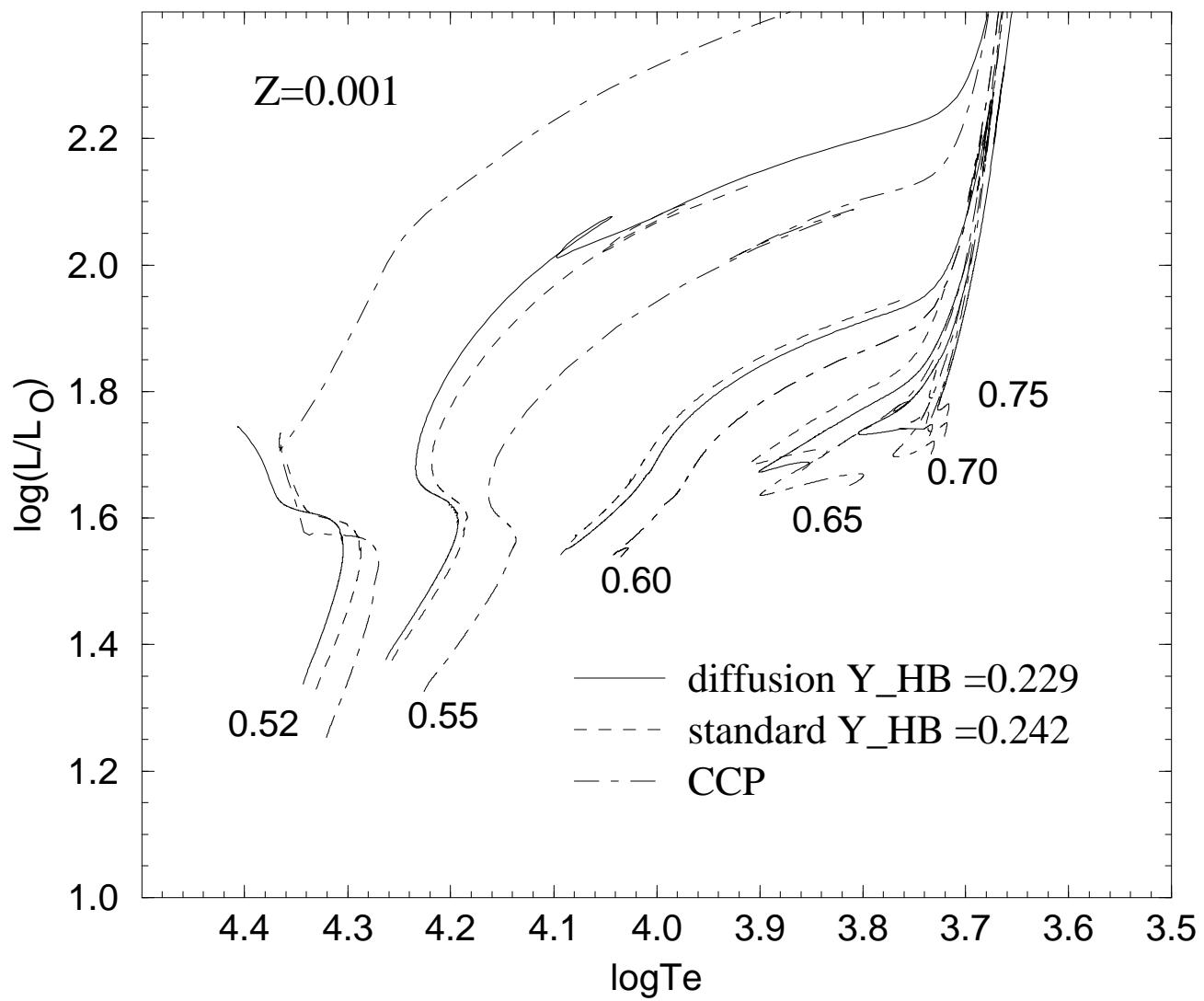


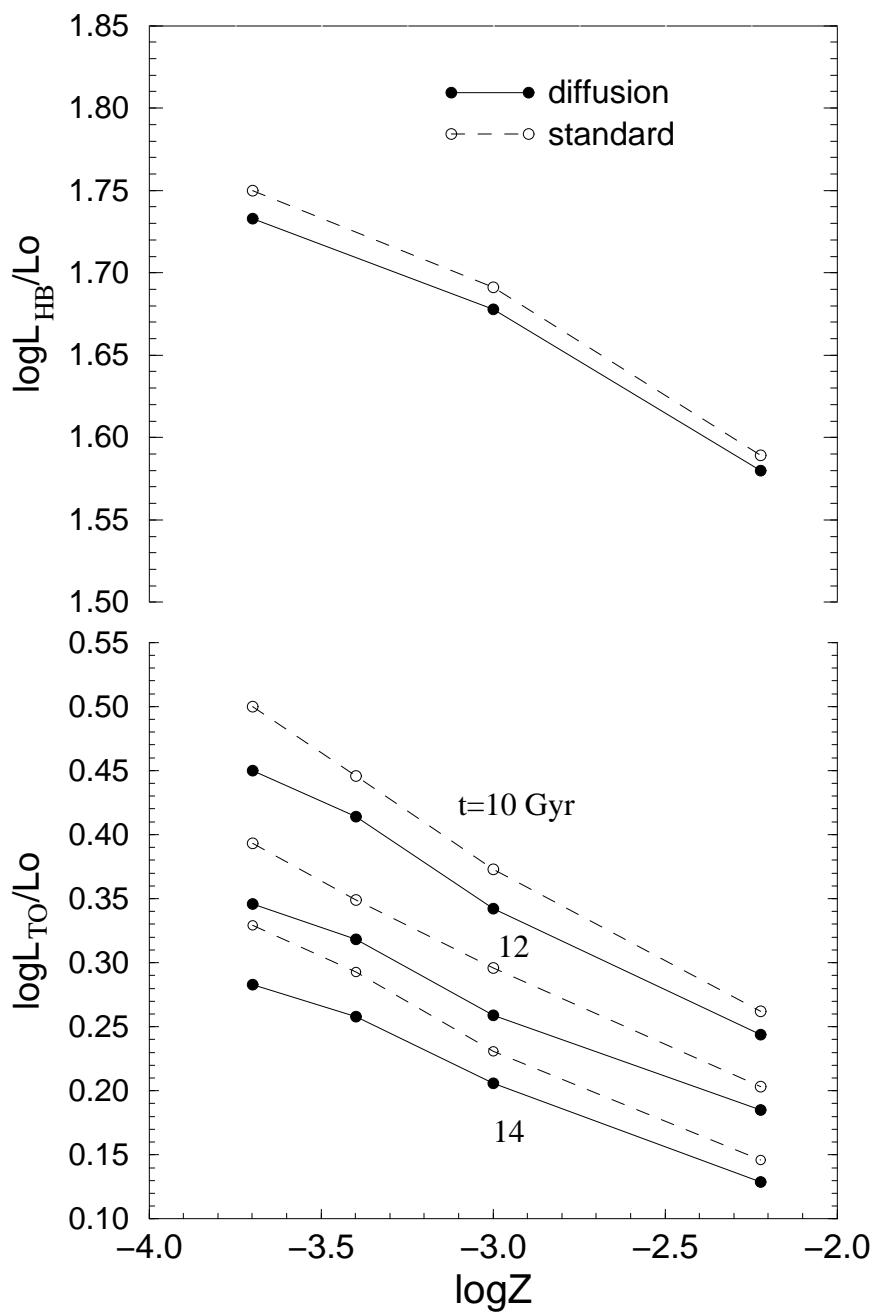


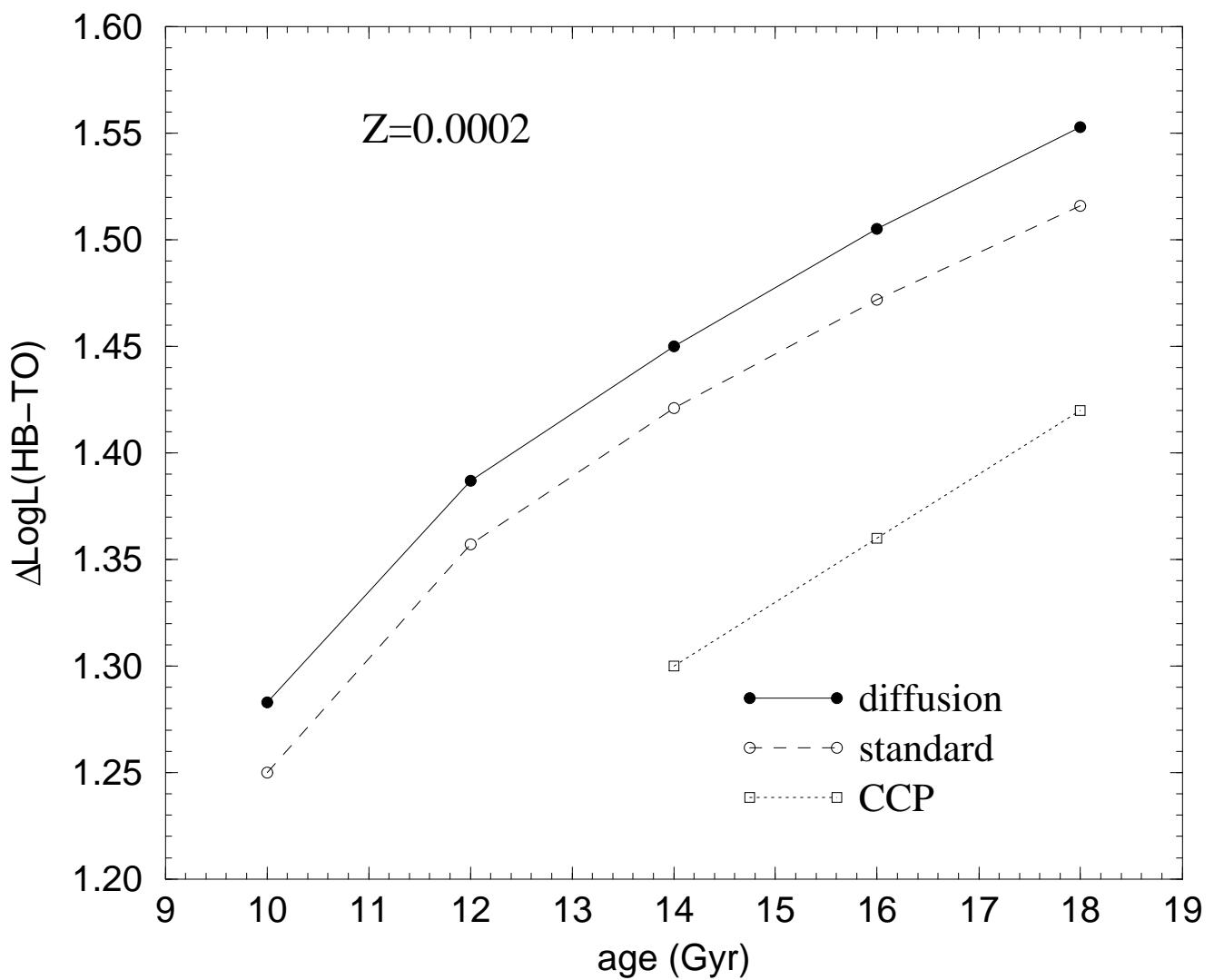


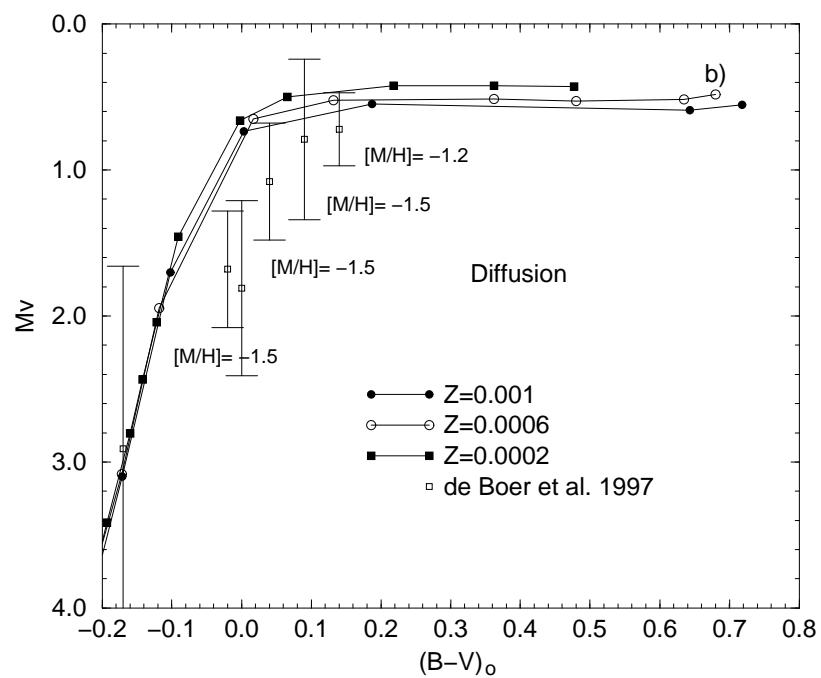
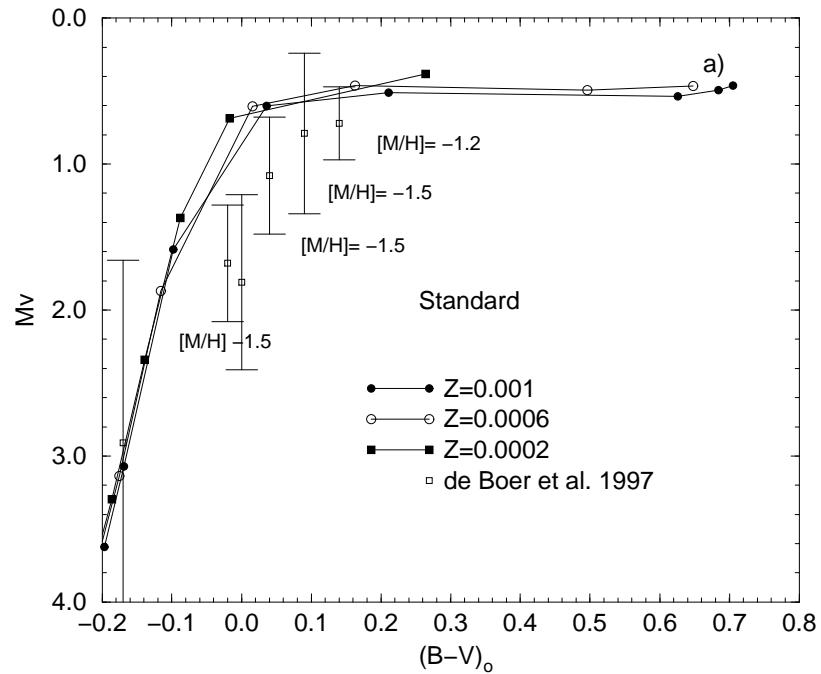












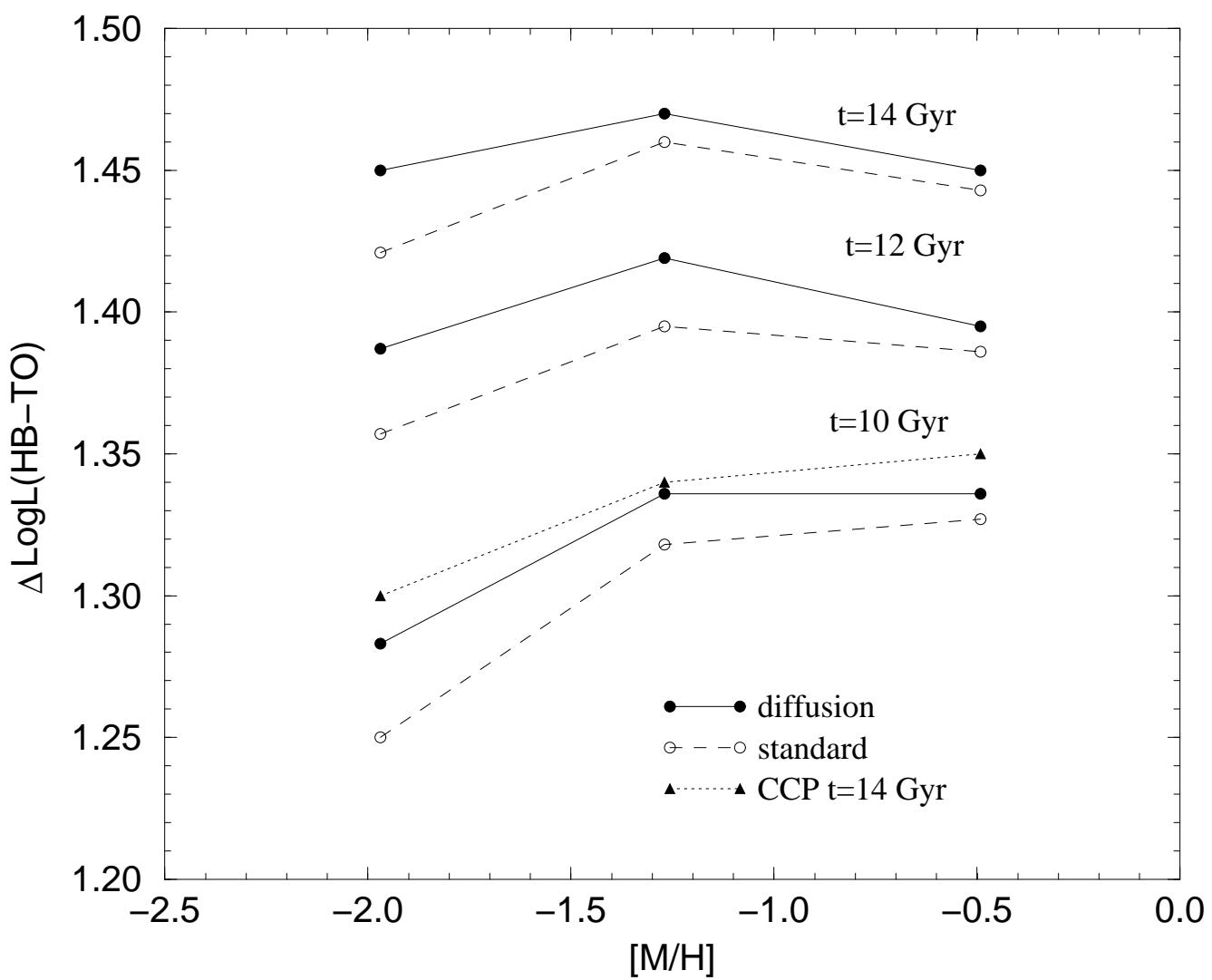


Table 1: Steps in the input physics and related selected evolutionary quantities for a $0.8 M_{\odot}$, $Y=0.23$, $Z=0.0001$ model, assumed as progenitor of the $0.7 M_{\odot}$ ($Y=0.238$) HB model at the bottom of the table. Ages for the $0.8 M_{\odot}$ model and for the $0.7 M_{\odot}$ HB model are, respectively, in Gyr and in Myr.

Step	1	2	3	4	5	6	7	8
EOS	Str88	Str88	Str88	OPAL	OPAL	OPAL	OPAL	OPAL
OPAC	LAOL	OPAL						
OPAC-CO	LAOL	LAOL	OPAL	OPAL	OPAL	OPAL	OPAL	OPAL
α -rates	Fow75	Fow75	Fow75	Fow75	Cau88	Cau88	Cau88	Cau88
NEU	Mun85	Mun85	Mun85	Mun85	Mun85	Haft94	Mun85	Haft94
LogL^{TO}	0.384	0.382	0.382	0.411	-	-	0.410	0.410
t^{TO}	13.62	13.50	13.50	11.58	-	-	11.58	11.58
LogL^{flash}	3.245	3.279	3.279	3.275	-	-	3.291	3.322
t^{flash}	15.28	15.17	15.17	13.06	-	-	13.22	13.22
M_c	0.5054	0.5098	0.5098	0.5054	0.5054	0.5054	0.5092	0.5152
Y_{HB}	0.238	0.238	0.238	0.238	0.238	0.238	0.238	0.238
LogL_{ZAHB}	1.617	1.635	1.635	1.656	1.663	1.663	-	1.666
LogTe_{ZAHB}	3.976	3.991	3.990	3.983	3.974	3.974	-	4.010
t_{HB}	96.97	93.71	87.17	87.65	80.44	80.28	-	75.02

Table 2: Evolutionary properties at the TO for step 8 ‘best’ models. The age at the track TO is in Gyr.

Z	M	t^{TO}	LogL^{TO}	LogTe^{TO}
0.0001	0.8	11.6	0.410	3.826
0.0002	0.6	33.3	-0.021	3.773
0.0002	0.7	18.6	0.188	3.797
0.0002	0.8	11.2	0.378	3.824
0.0002	0.9	7.4	0.572	3.859
0.0002	1.0	5.0	0.723	3.899
0.001	0.6	35.6	-0.178	3.755
0.001	0.7	20.0	0.060	3.777
0.001	0.8	11.7	0.231	3.799
0.001	0.9	7.4	0.393	3.822
0.001	1.0	5.0	0.577	3.852
0.006	0.65	38.3	-0.250	3.729
0.006	0.7	29.1	-0.140	3.740
0.006	0.8	16.8	0.017	3.759
0.006	0.9	9.5	0.134	3.777
0.006	1.0	5.5	0.263	3.796

Table 3: Selected TO quantities for H burning isochrones for the labeled values of metallicity and age. The age at the isochrone TO is in Gyr.

Z	t^{TO}	LogL^{TO}	LogTe^{TO}	M^{TO}
0.0002	8	0.595	3.853	0.877
	9	0.547	3.843	0.859
	10	0.500	3.835	0.834
	11	0.457	3.827	0.812
	12	0.393	3.821	0.790
	13	0.357	3.817	0.773
	14	0.329	3.813	0.758
	15	0.303	3.809	0.744
	16	0.278	3.805	0.731
	17	0.255	3.802	0.719
0.001	18	0.234	3.799	0.709
	8	0.455	3.820	0.897
	9	0.411	3.814	0.869
	10	0.373	3.809	0.845
	11	0.339	3.804	0.824
	12	0.309	3.799	0.806
	13	0.256	3.796	0.786
	14	0.231	3.793	0.772
	15	0.208	3.790	0.758
	16	0.187	3.787	0.746
0.006	17	0.167	3.784	0.734
	18	0.148	3.782	0.723
	9.5	0.281	3.781	0.932
	10	0.262	3.779	0.921
	11	0.229	3.775	0.900
	12	0.203	3.772	0.881
	13	0.167	3.769	0.862
	14	0.146	3.767	0.847
	15	0.114	3.764	0.831
	16	0.096	3.762	0.819
0.006	17	0.079	3.760	0.807
	18	0.059	3.757	0.796

Table 4: Selected evolutionary quantities for $0.8 M_{\odot}$ Red Giant models without diffusion. The age at the He flash is in Gyr.

Z	M_c	Y_{HB}	t^{flash}	LogL^{flash}	LogL^{bump}
0.0001	0.515	0.238	13.22	3.322	2.245
0.0002	0.511	0.239	13.22	3.349	2.168
0.001	0.503	0.242	14.52	3.396	1.943
0.006	0.494	0.246	20.89	3.442	1.535

Table 5: Selected evolutionary quantities for ZAHB models without diffusion. The time spent during the central helium burning phase (in Myr) is compared with results from CCP.

Z=0.0001 Y _{HB} =0.238 M _c =0.515 M _⊙				
M	LogL	LogTe	t _{He}	t _{He} ^{CCP}
0.53	1.357	4.410	93.0	-
0.55	1.395	4.320	88.2	113.6
0.60	1.471	4.217	82.6	105.3
0.65	1.582	4.108	78.5	100.6
0.70	1.662	4.010	75.0	96.9
0.75	1.727	3.926	74.4	94.7
0.77	1.744	3.900	73.9	-
0.80	1.769	3.860	-	91.3

Z=0.001 Y _{HB} =0.242 M _c =0.503 M _⊙				
M	LogL	LogTe	t _{He}	t _{He} ^{CCP}
0.53	1.328	4.331	98.8	-
0.55	1.373	4.257	94.2	119.9
0.60	1.562	4.084	86.6	111.2
0.65	1.688	3.855	84.0	106.8
0.70	1.747	3.739	82.5	104.3
0.75	1.777	3.726	81.6	101.9
0.80	1.794	3.722	-	-

Table 6: Selected evolutionary quantities for models with diffusion at the track TO. The age at the track TO is in Gyr.

Z	M	t^{TO}	LogL^{TO}	LogTe^{TO}
0.0002	0.6	30.8	-0.078	3.751
	0.7	17.7	0.140	3.786
	0.8	10.9	0.330	3.814
	0.9	7.1	0.536	3.848
	1.0	4.9	0.702	3.890
0.001	0.7	18.7	0.026	3.770
	0.8	10.9	0.187	3.794
	0.9	6.7	0.348	3.818
	1.0	4.8	0.551	3.847
0.006	0.65	35.1	-0.255	3.721
	0.70	26.6	-0.172	3.733
	0.80	15.4	-0.027	3.754
	0.90	8.7	0.101	3.774
	1.0	5.1	0.245	3.794

Table 7: Selected TO quantities for H burning isochrones with diffusion for the labeled assumptions about age and original chemical composition. The age at the isochrone TO is in Gyr.

Z=0.0002 Y=0.23			
t^{TO}	LogL^{TO}	LogTe^{TO}	M^{TO}
8	0.557	3.840	0.881
9	0.501	3.831	0.852
10	0.450	3.822	0.826
11	0.406	3.814	0.804
12	0.346	3.809	0.783
13	0.313	3.804	0.766
14	0.283	3.800	0.750
15	0.255	3.796	0.736
16	0.228	3.793	0.723
17	0.204	3.789	0.711
18	0.180	3.786	0.700
Z=0.001 Y=0.23			
t^{TO}	LogL^{TO}	LogTe^{TO}	M^{TO}
8	0.429	3.814	0.888
9	0.383	3.807	0.861
10	0.342	3.801	0.837
11	0.305	3.796	0.816
12	0.259	3.791	0.795
13	0.232	3.788	0.779
14	0.206	3.784	0.764
15	0.183	3.781	0.750
16	0.160	3.778	0.737
17	0.140	3.775	0.726
18	0.120	3.773	0.715
Z=0.006 Y=0.23			
t^{TO}	LogL^{TO}	LogTe^{TO}	M^{TO}
9.5	0.265	3.776	0.926
10	0.244	3.774	0.914
11	0.204	3.770	0.891
12	0.185	3.767	0.874
13	0.156	3.763	0.856
14	0.129	3.760	0.840
15	0.090	3.757	0.823
16	0.067	3.755	0.809
17	0.047	3.752	0.797
18	0.031	3.750	0.785

Table 8: Selected evolutionary quantities for stellar models with element diffusion at the He flash. The age at the He flash is in Gyr.

M	M_c	Y_{HB}	t^{flash}	LogL^{flash}	LogL^{bump}
Z=0.0002					
0.7	0.5188	0.2144	20.33	3.359	2.056
0.8	0.5148	0.2261	12.77	3.352	2.139
0.9	0.5113	0.2344	8.51	3.344	2.209
1.0	0.5081	0.2402	5.96	3.334	2.290
Z=0.001					
0.7	0.5107	0.2178	22.24	3.410	1.825
0.8	0.5068	0.2294	13.98	3.404	1.917
0.9	0.5041	0.2377	9.28	3.398	1.983
1.0	0.5018	0.2434	6.46	3.393	2.043
Z=0.006					
0.7	0.5034	0.2184	31.36	3.460	1.399
0.8	0.5001	0.2308	19.91	3.457	1.495
0.9	0.4975	0.2392	13.30	3.457	1.578
1.0	0.4958	0.2446	9.20	3.530	1.657

Table 9: Selected evolutionary quantities for HB models with diffusion. The time spent during the central He burning phase is in Myr.

Z=0.0002 Y _{HB} =0.226 M _c =0.515 M _⊙			
M	LogL	LogTe	t _{He}
0.53	1.352	4.395	-
0.54	1.369	4.352	-
0.55	1.386	4.318	86.6
0.58	1.437	4.237	-
0.60	1.478	4.192	78.1
0.62	1.524	4.145	78.0
0.65	1.593	4.072	76.0
0.70	1.678	3.952	73.9
0.72	1.703	3.909	-
0.75	1.734	3.849	72.9
0.77	1.751	3.813	-
0.80	1.773	3.772	66.6

Z=0.001 Y _{HB} =0.229 M _c =0.507 M _⊙			
M	LogL	LogTe	t _{He}
0.53	1.337	4.343	95.0
0.55	1.376	4.263	92.8
0.60	1.540	4.095	84.8
0.65	1.671	3.866	81.4
0.70	1.730	3.735	80.3
0.75	1.760	3.719	78.7

Table 10: Theoretical estimates of the parameter R. Times are in Myr.

Z	0.0002	0.0002	0.001	0.001	0.006	0.006
Diffusion	NO	YES	NO	YES	NO	YES
LogL _{ZAHB}	1.759	1.744	1.701	1.687	1.594	1.574
t _{He}	76.16	71.33	83.70	81.13	93.00	91.30
t _{RG}	73	77.24	79.52	85.22	68.3	83.20
R(3.83)	1.043	0.923	1.053	0.952	1.362	1.097
R(3.83)+0.05	1.142	1.010	1.395	1.034	1.525	1.203
R(3.83)+0.10	1.248	1.102	1.522	1.120	1.690	1.308